Math 284   Name:_____
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Practice Exam 1

You have to show your work reducing a matrix for Problem #1 only. For all other problems, you may use the rref() command in your calculator. Write down any reduced matrix obtained from the rref() command.

1) Use elementary row operations to transform the matrix into reduced row echelon form. Give a list of the operations you used to do this, and show what the matrix looks like after each column is cleared.

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

2) Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$.

    a) Find $3\mathbf{u}$.

    b) Find $\dfrac{1}{3}\mathbf{v}$.

    c) Find $4\mathbf{u} - 3\mathbf{v}$.

3) Determine if the following statements are true or false. Justify your answer.

    a) In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

    b) The set Span$\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.

    c) If A is *mxn* and if the equation A$\mathbf{x}$=b is inconsistent for some $\mathbf{b}$ in $\mathbb{R}^m$, then A cannot have a pivot in every row.

    d) The homogeneous equation A$\mathbf{x}$=$\mathbf{0}$ has the trivial solution if an only if the equation has at least one free variable.

    e) The columns of any 4x5 matrix are linearly independent.

4) Calculate the following, if possible.

a) $\begin{bmatrix} -2 & 1 & 4 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 3 & 4 \\ -2 & 4 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

5) Let $\mathbf{a}_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 16 \\ 26 \end{bmatrix}$.

a) Determine whether **b** is a linear combination of **a₁** and **a₂**. If it is, find appropriate coefficients.

b) Is **b** in the Span{**a₁**, **a₂**}?

6) Let $A = \begin{bmatrix} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{bmatrix}$ be the augmented matrix for a system of equations.

Determine for what values of h the system is consistent.

7) Do the vectors $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$ span $\mathbb{R}^3$?

8) Determine whether the columns of the following matrices are linearly independent or linearly dependent. Justify your answers.

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & -6 \\ -3 & 9 \\ 1 & 3 \\ -2 & -6 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 6 & 9 \\ 1 & 5 & 9 \\ 2 & 2 & 2 \end{bmatrix}$

9) Let $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 7 \\ -6 \end{bmatrix}$.

a) Find the general solution to the equation $A\mathbf{x} = \mathbf{0}$. Write your answer in parametric vector form.

b) Find the general solution to the equation $A\mathbf{x} = \mathbf{b}$. Write your answer in parametric vector form.

10) It is possible to convert Toluene (C_7H_8) and oxygen (O_2) to produce carbon suboxide(C_3O_2) and water (H_2O). Create the matrix corresponding to this equation and balance the chemical equation, using the smallest whole number solution:

$$\_\_\_ C_7H_8 + \_\_\_ O_2 \rightarrow \_\_\_ C_3O_2 + \_\_\_ H_2O$$